PHYSICAL REVIEW B VOLUME 2, NUMBER 11 1 DECEMBER 1970

Response of Highly Damped Josephson Junctions to External, Low-Frequency Noise Currents*

H. KANTER AND F. L. VERNON, JR.

The Aerospace Corporation, El Segundo, California 90045

(Received 11 May 1970)

The dc characteristics of superconducting point-contact junctions subjected to external noise currents are measured under the condition that the noise-frequency spectrum is below the Josephson frequency. It is shown that the curves are in agreement with a model based on modulation of the noise-free dc characteristic. Although in the low-bias-voltage region the shape of the curves is qualitatively similar to previous theoretical predictions for the case of high-frequency noise, for higher voltages the shapes are different. Quantitative differences exist over the whole range of bias voltage.

Recently, several papers have appeared[1–3] on the effect of thermal fluctuations on the dc I-V characteristics of Josephson junctions. In these theories the phase difference between superconductors is treated like the spatial coordinate of a particle subjected to collisions with a gas in thermal equilibrium. Families of I-V characteristics were obtained with parameters depending on temperature and the response time associated with the resistive and capacitive portion of the junction. Anderson and Goldman[4] reported experimental results which were compared with theory. However, to obtain agreement in the region near zero voltage, the assumption of a temperature of 10°K compared with an experimental value of about 3.8°K was required, suggesting noise currents of other than thermal origin. We find that the current-voltage characteristics resulting from the application of low-frequency noise currents are very similar in shape to those for the high-frequency case at low values of bias voltage.[5] For "slow" noise the response is the result of voltage averaging over the static I-V characteristic of the junction and is different from the theoretical predictions for thermal excitation. The similarity of the response to slow noise in a typical experimental setup complicates verification of the predicted temperature dependence. In what follows we demonstrate the effects of low-frequency random excitation in comparison with the theoretical results for rapid fluctuations.

For a current-driven Josephson junction small enough to have uniform current density across the contact area, the equation of motion is

$$C_1(dV/dt)+(V/R_1)+I_c\sin\varphi-I_B=I(t), \quad (1)$$

where C_1 is the capacitance, R_1 the "normal" resistance of the junction, I_c the critical current, and φ the phase difference of the order parameter across the junction. I_B is the dc battery current, and $I(t)$ a fluctuating noise current.

With the Josephson condition $d\varphi/dt=2eV/\hbar$, the equation can be rewritten in the phase difference φ only. For a sufficiently small value of the product R_1C_1 the inertia term may be neglected and in case $I(t)=0$ an analytical solution for the average phase speed is possible as has been shown previously.[6–8] The I-V relation is described by $(V/V_0)^2=(I_B/I_C)^2-1$ for $I_B/I_c>1$ and $V/V_0=0$ for $I_B/I_c\leq 1$. The hyperbola is shown in Fig. 1 by the solid line closely followed by the solid dots. (The current scale is in units of the critical current, the voltage in units $V_0=R_1I_c$.)

If $eV\ll kT$, the driving term of Eq. (1) fluctuates on a time scale short compared to the inverse Josephson frequency $\nu^{-1}=h/2eV$. The noise currents may then be considered of thermal origin with $\langle I(t+\tau)I(\tau)\rangle=kT\delta(\tau)/R_1$, the Johnson noise of the resistive portion of the circuit. In case $\Omega=R_1C_1\omega_J\ll 1$ [where $\omega_J=(2eI_c/\hbar C_1)^{1/2}$, the Josephson plasma frequency], the Fokker-Planck equation corresponding to Eq. (1) has been solved (Refs. 1 and 2) and I-V characteristics obtained as shown by the dashed lines in Fig. 1 for various values of $\gamma=\hbar I_c/ekT$, which is proportional to the ratio of pair potential in the tunneling gap to thermal energy.[9] We note that at all current levels, the theory indicates that addition of noise current *increases* the average voltage across the junction. For the more general case $\Omega\gtrsim 1$, Eq. (1) was numerically integrated in Ref. 3 by simulating the driving term with pulses distributed in amplitude and time around mean values corresponding to collisions with an ideal gas of light particles. In this case the noise-modified curves cross the noise-free curves and approach them from above, i.e., noise currents can decrease the average voltage across a junction. Since in what follows we consider only characteristics with $\Omega\ll 1$, the theoretical results for $\Omega\gtrsim 1$, are not included in Fig. 1.

We computed the I-V characteristic to be expected when driving a junction of $\Omega\ll 1$ with noise current fluctuating on a time scale long compared to the period ν^{-1}. The instantaneous voltage-current relation of the junction is then simply given by the static I-V characteristic.[10] However, the voltmeter with time constant R_2C_2 (see inset of Fig. 1) averages over the instantaneous junction voltage if $(R_2C_2)^{-1} \ll \nu_N$, the frequency limit of the noise current. The average indicated voltage for a slowly varying noise current of Gaussian amplitude distribution around a certain battery current I_B can thus be computed with

$$\frac{V}{V_0}=\frac{1}{(2\pi)^{1/2}}\frac{1}{I_n}\left\{\int_{I_c}^{\infty}\left[\left(\frac{I}{I_c}\right)^2-1\right]^{1/2}\exp\left(-\frac{(I-I_B)^2}{2I_n^{\,2}}\right)dI\right.$$
$$\left.-\int_{-\infty}^{-I_c}\left[\left(\frac{I}{I_c}\right)^2-1\right]^{1/2}\exp\left(-\frac{(I-I_B)^2}{2I_n^{\,2}}\right)dI\right\},\quad(2)$$

where I_n is the root-mean-square noise-current amplitude. The result is shown in Fig. 1 by the solid lines. Particularly for small voltages, $V/V_0 \ll 1$, the sets of curves representing response to "fast" and "slow" noise currents (dashed and solid curves, respectively) are very similar in shape.[11] However, here the noise-modified curves cross the noise-free characteristic for $\Omega \ll 1$, which in the thermal case was only obtained for $\Omega \gtrsim 1$. This is, of course, simply a consequence of the convex curvature of the static I-V characteristic.

We were able to correlate the slow noise response experimentally on a current-driven point contact with a noise-free characteristic whose shape is very nearly that expected theoretically for $\Omega \ll 1$. The experimental characteristic without external noise applied is indicated by the solid dots in Fig. 1. Only at $V/V_0 > 1$ a slight deviation from the "ideal" hyperbola is observed. The slight depression of I for $V/V_0 \sim 0$ is due to internal noise currents, and is considerably reduced by cooling the junction to 1.5°K. The fact that no hysteresis appeared at this temperature is in agreement with theory when $\Omega \ll 1$. Battery current and noise current from a General Radio type 1398B generator were applied to the junction through a resistance of 10 KΩ (R_0 in Fig. 1), which is larger by a factor of at least 20 than the largest dynamic resistance encountered in the experimental junctions. This ensured operation in the current-driven mode. The voltmeter time constant was of the order of 10^{-1} sec. The measured curves taken with an upper noise-frequency limit of 20 kHz (for which the generator produced a nearly Gaussian amplitude distribution) are indicated by the open circles in Fig. 1. The shape clearly follows that of the slow noise curves represented by the solid lines. For each noise amplitude shown a crossover is observed. The experimentally obtained dependence of voltage V at constant battery current on noise amplitude follows exactly that expected from the slow noise curves.

In conclusion, these results show that the power spectrum of the noise current through a Josephson point-contact junction relative to its rotational frequency is of decisive importance in determining the exact dc response. Low-frequency noise, particularly if it arises from filtered high-temperature external circuitry, may dominate the high-frequency thermal noise generated in the junction in affecting the I-V characteristic. Consequently, the postulation of an effective junction noise temperature to account for the dc response characteristics must be considered with caution. Since a clear distinction between the response to slow and fast noise currents is possible only at either relatively large sample voltages or relatively small noise currents (large γ) comparison with experiment should emphasize these regions. The deviation from theoretical prediction observed by Simmonds and Parker[12] for $\gamma > 15$ may at least in part have its origin in slow noise currents as discussed here.

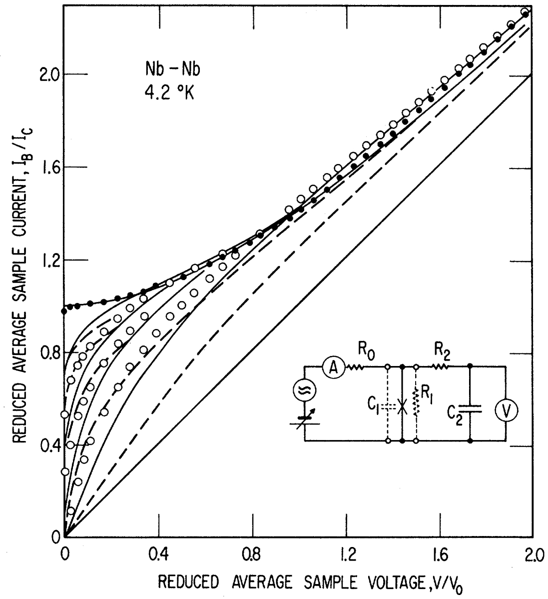

FIG. 1. Junction characteristic subject to noise currents. The solid straight line indicates the resistive contribution. The solid line followed closely by the solid circles is the response of an ideal, heavily damped, noise-free junction. The remainder of solid lines indicates the response of an ideal junction to various slow noise-current amplitudes with parameter $I_n/I_c = 0.2$, 0.4, 0.6, and 1 in going from left to right, respectively. Closed and open circles represent experimental results for which $I_n/I_c = 0$, 0.33, 0.54, and 0.8, respectively. The dashed lines are theoretical results of Ref. 2. The parameter is $\Omega = 0$ and $\gamma = 40$, 20, 10, 5, and 2, respectively ($V_0 = 178\ \mu$V, $I_c = 23\ \mu$A).

Note added in proof. Since this paper was submitted, a further experimental study[13] of the effect of noise on the dc I-V characteristic of small tunnel junctions appeared purporting to support the results of Ref. 1. External noise currents with a maximum frequency of 5 MHz were used to modify the characteristics measured on a voltage scale in excess of microvolts. The noise frequencies were thus considerably below the Josephson frequencies for which the characteristics were measured. The fitted parameter in these studies was $\gamma \leq 3$. For values of γ in this range, little quantitative distinction between "slow" and "fast" noise exists. Thus, none of these three experiments[4,12,13] can be considered proof of the validity of thermal theory.

[*] Work supported by the U.S. Air Force under Contract No. F 0470169-C-0066.

[1] Yu. H. Ivanchenko and L. A. Zil'berman, Zh. Eksperim. i Teor. Fiz. 55, 2395 (1968) [Soviet Phys. JETP 28, 1272 (1969)].

[2] V. Ambegaokar and B. I. Halperin, Phys. Rev. Letters 22, 1364 (1969).

[3] J. Kurkijarvi and V. Ambegaokar, Phys. Letters 31A, 314 (1970).

[4] J. T. Anderson and A. M. Goldman, Phys. Rev. Letters 23, 128 (1969).

[5] Low frequency or slow noise in the case considered here refers to a noise-frequency spectrum which is much lower than the Josephson frequency but higher than the inverse response time of the voltmeter used to trace the I-V characteristic.

[6] W. C. Stewart, Appl. Physics Letters 12, 277 (1968).

[7] D. E. McCumber, J. Appl. Phys. 39, 3113 (1968).

[8] A. Th. A. M. DeWaele and R. De Bruyn Ouboter, Physika 41, 225 (1969).

[9] See, for instance, B. D. Josephson, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), p. 423.

[10] Actually, the noise current is assumed to fluctuate on a time scale long compared to $(\nu^*)^{-1}$, where ν^* corresponds to voltages $V^* < V$ which contribute appreciably to the average observed voltage V.

[11] In fact both sets of curves are expected to coincide for $V \to 0$ since for finite noise frequency eventually $2eV/h < \nu_N$.

[12] M. Simmonds and W. H. Parker, Phys. Rev. Letters 24, 876 (1970).

[13] S. A. Buckner, J. T. Chen, and D. N. Langenberg, Phys. Rev. Letters 25, 738 (1970).

PHYSICAL REVIEW B VOLUME 2, NUMBER 11 1 DECEMBER 1970

Kondo Effect in La$_{1-x}$Ce$_x$ Alloys under Pressure*

K. S. Kim and M. B. Maple

Institute for Pure and Applied Physical Sciences, University of California, San Diego, La Jolla, California 92037

(Received 6 July 1970)

The low-temperature electrical resistance of La$_{1-x}$Ce$_x$ alloys in both the fcc and the dhcp phase has been measured under pressure. Between normal pressure and 19 kbar the resistance always exhibited a minimum at T_{min}, which initially increased slightly under pressure, but remained constant above 7 kbar. The resistance $R(T)$ for $T < T_{\mathrm{min}}$ varied as $\ln T$ down to the superconducting transition temperature T_c or the limiting measuring temperature (1.3°K). The slope, $-dR(T)/d \ln T$, varied appreciably and nonmonotonically with pressure; the relationship between the depression of T_c and $-dR(T)/d \ln T$ as a function of pressure is discussed.

Previously we reported minima in the variation of the superconducting transition temperature (or maxima in the pairbreaking parameter) of La$_{3-x}$Ce$_x$In [1] and La$_{1-x}$Ce$_x$ [2] alloys with pressure. For the La$_{1-x}$Ce$_x$ alloys at pressures above 100 kbar, the depression $\Delta T_c = T_{c0} - T_c$ is more than an order of magnitude smaller than at maximum pair breaking ($\sim$15 kbar) and at least five times smaller than at normal pressure. Here T_{c0} is the superconducting transition temperature of the host metal and T_c is that of the alloy. From this it was inferred that the Ce 4f level moves toward the Fermi level upon the application of pressure, giving rise to an initial increase of $|J_{\mathrm{eff}}|$, the conduction electron-impurity spin exchange coupling strength, and at sufficiently high pressure, to a transition of the Ce impurities from a magnetic to a nonmagnetic state.

Demagnetization of the Ce impurities was suggested[2] to proceed within the context of the Friedel-Anderson model.[3] The spin-up and spin-down sublevels, split below and above the Fermi level by intra-atomic Coulomb repulsion at low pressure, become degenerate and nonmagnetic at high pressure when the spin-up sublevel begins to significantly overlap the Fermi level. On the other hand, it has been suggested that a continuous increase of the Kondo temperature (T_K) with pressure could provide an alternative explanation for the pair-breaking maxima.[1,4] Both Zuckermann[5] and Müller-Hartmann and Zittartz[4] (MZ) have shown that the depression of T_c as a function of $\ln T_K/T_{c0}$ exhibits a maximum which occurs, in the MZ calculation, when $T_K \sim 12 T_{c0}$. In an attempt to determine how the Kondo temperature of La$_{1-x}$Ce$_x$ alloys depends upon pressure, and hence decide whether a magnetic-nonmagnetic transition or a continuous increase of T_K is responsible for the maximum and subsequent decrease in pair breaking with pressure, we have measured the low-temperature electrical resistance of La$_{1-x}$Ce$_x$ alloys under pressure to $\sim$19 kbar.

Samples of La$_{1-x}$Ce$_x$ were prepared by melting the constituents under argon in a conventional arc furnace. The resultant ingots were then converted to the dhcp phase by cold-rolling them into foils $\sim$0.1 mm in thickness, which were subsequently annealed in vacuum at 200°C for 3 h. To obtain the fcc phase, unannealed cold-rolled foils were heat treated in vacuum at 600°C for 10 h and then rapidly quenched in water. The agreement of the superconducting transition temperatures with the previous results[2] indicated that the right phases were obtained. A Be-Cu clamp was used to generate pressures up to 19 kbar and a Teflon bucket with a Be-Cu cap was used to contain the pressure transmitting liquid (1:1 mixture of isoamyl alcohol and n-pentane), the sample, the leads thereof, and a superconducting Pb manometer. A detailed description of the pressure seal is given elsewhere.[6] Leads were attached to the samples by spot welding and the resistance was measured by means of a standard four-lead dc technique.